Harmonic Oscillator (mass-spring-damper system)

$$\frac{d^2 y}{dt^2} + c\frac{dy}{dt} + k^2 y = F(t)$$

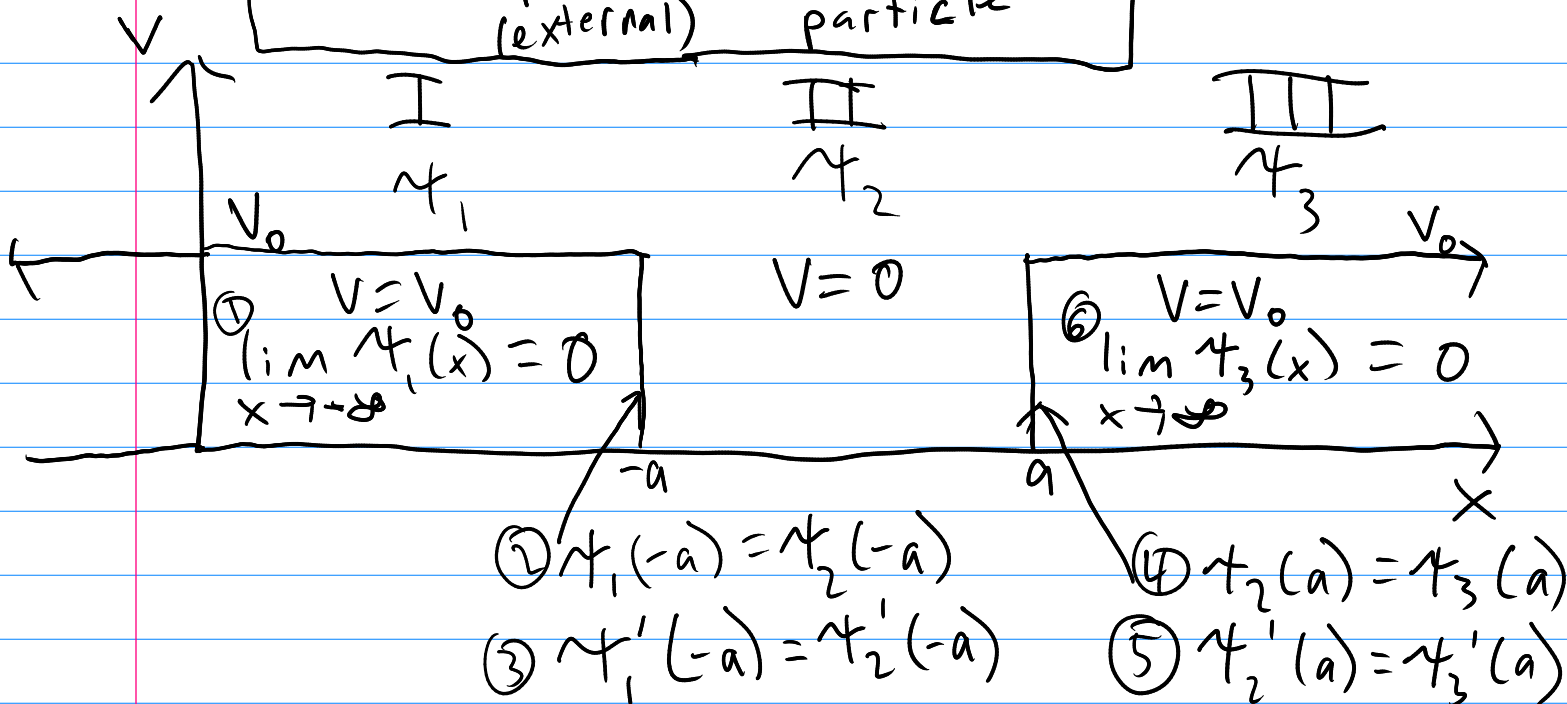$\underbrace{\hspace{2cm}}_{\text{forcing function}}$

initial position: $y(0) = y_0$
initial velocity: $y'(0) = V_0$

Time-independent Schrodinger equation: Finite potential well

$$\boxed{\frac{-d^2 \Psi}{dx^2} + V\Psi = E\Psi}$$

$\underbrace{}_{\substack{\text{potential} \\ \text{(external)}}}$ $\underbrace{}_{\substack{\text{energy of the} \\ \text{particle}}}$

assume
$E < V_0$

I $\quad \Psi_1$ II $\quad \Psi_2$ III $\quad \Psi_3$

V_0 $\qquad V=0 \qquad$ V_0

① $V = V_0$
$\lim\limits_{x \to -\infty} \Psi_1(x) = 0$

⑥ $V = V_0$
$\lim\limits_{x \to \infty} \Psi_3(x) = 0$

$-a \qquad a$

② $\Psi_1(-a) = \Psi_2(-a)$
③ $\Psi_1'(-a) = \Psi_2'(-a)$

④ $\Psi_2(a) = \Psi_3(a)$
⑤ $\Psi_2'(a) = \Psi_3'(a)$

The Scrödinger equation applies in each region.
Apply BC's to obtain transcendental equation
for allowed (quantized) energy levels, E:

$$-2\left[E(V_0 - E)\right]^{\frac{1}{2}} \cos(2a\sqrt{E}) + \left[2E - V_0\right]\sin(2a\sqrt{E}) = 0$$